An Extended Horizon Feedback/ Feedforward Self-Tuning Controller

A feedforward/feedback version of a single input/single output self-tuning controller has been developed and tested both via simulation studies and experiments on a section of an absorption/desorption pilot plant. The algorithm is based on the recursive least squares estimation of parameters for the linear models relating the output to the controlled input and to the disturbances; adaptation is achieved using a variable forgetting factor. The control input at each time interval is calculated using one of several single-step extended-horizon control strategies.

The results show that the performance of the algorithm is insensitive to the choice of initial parameters, all of which have a readily identifiable intuitive basis. The algorithm is especially robust against deterministic disturbances (measurable and unmeasurable) and unknown and varying time delays. Computational load beyond that of a feedback-only version is minimal.

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Introduction

In principle, feedforward control has substantial advantages over feedback control since the former can compensate for the effects of disturbances before they have had a chance to alter the behavior of a plant. To be effective, however, classical feedforward control requires the *a priori* knowledge of a good process model. Furthermore, feedforward often calls for drastic control action when there are large input disturbances; there is no universal way to detune this control action. For these reasons, feedforward control is used only sparingly in practical situations.

The prerequisite of a model for feedforward control, however, can be satisfied rather easily in adaptive self-tuning control algorithms, since the parameters of a simplified model relating the disturbances to the output can be estimated on-line if the output and disturbances are both measurable. This estimation can be carried out alongside that of a model relating the controlled input to the output, as is done in standard feedback adaptive control algorithms. The inclusion of feedforward action can actually improve the robustness of a purely feedback adaptive controller; the latter will often overreact to the effects of large disturbances by changing model parameters excessively and increasing the estimated gain of the controller, thereby deteriorating closed-loop performance (Morris et al., 1982; Hiram et al., 1986; Lim et al., 1987). The feedforward algorithm can distinguish between a disturbance and a change in the process model and, hence, can react more appropriately.

There are several different ways of designing adaptive feedforward controllers. One of them, based upon optimal control theory (Peterka, 1982; Sternard, 1986; Hunt et al., 1987) requires substantial computational effort and is relatively unpopular among practitioners. Simulation results, however, do show a considerable improvement in performance when adaptive feedforward is included. Simpler feedforward algorithms are presented by Schumann and Christ (1979); they adapt quickly and improve closed-loop performance but require the knowledge of model order and time delays.

In an alternative approach, a feedback/feedforward version of the generalized minimum-variance self-tuning algorithm (Clarke and Gawthrop, 1975) was tested both with nonlinear simulations and experiments on a pilot-scale distillation column (Morris et al., 1985). It was shown that, most of the time, the adaptive feedforward action significantly improved the performance of the controlled systems, especially in reducing the transient response time. The algorithm, however, was sensitive to the

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selection of model order and to varying and unknown time-delays; in addition, the feedforward action was too sensitive under certain conditions and required some detuning. Allidina et al. (1981) proposed a new algorithm which allowed such detuning. Implementation in the temperature control loop of a pilot-scale nuclear reactor showed that the use of adaptive feedforward control greatly improved the closed-loop performance. The proposed controller, however, required a proper selection of the detuning factor of the feedforward action, a fact that detracts from practical implementations (Gawthrop, 1982).

A similar algorithm which incorporated pole assignment was proposed by Park (1986), and was tested with simulations in a ninth-order nonlinear model of a nuclear power plant. Since 1981, an algorithm with a fixed feedback control and a weighted adaptive feedforward control action has been employed in the Hungarian power system to improve load-frequency control (Vajk et al., 1985).

Another alternative is in the use of an extended-horizon control law (Ydstie, 1982), in which the control objective is set at a time T steps in the future, where T is equal to or larger than the maximum expected control delay time. The control action can be easily detuned by increasing the time horizon T. A generalization of this and other concepts has led to the Generalized Predictive Control algorithm of Clarke et al. (1987); some feedforward applications of that controller have been reported recently by Lambert (1987) and Clarke (1988).

The algorithm proposed here is an extension of the robust self-tuning regulator of Ydstie et al. (1985), for the case of measurable disturbances. An extended-horizon feedback/feedforward self-tuning regulator is obtained which is able to detune the feedforward control action without the need for an extra tuning parameter in the controller. In order to avoid offset and to improve the numerical properties of the estimator, the proposed self-tuner is expressed in incremental form (Fortescue, 1977; Clarke et al., 1983; Pérez-Correa, 1987). The main advantages of the controller presented here are its robustness against variable and unknown time delays, the ease of initial parameter selection, the simplicity of the feedforward calculation, and the possibility of detuning the control action in a natural way. Results are presented for several simulations and for experiments on a pilot-scale absorption/desorption plant. The advantages of the incorporation of feedforward into the algorithm far outweigh the modest increase in computation required.

Theory

It is assumed that the process to be controlled can be described by the following incremental model which includes a measurable disturbance:

$$A(z^{-1})\Delta y_k = z^{-du} B(z^{-1})\Delta u_k + z^{-dw} C(z^{-1})\Delta w_k + \Delta e_k \quad (1)$$

where the symbols z^{-1} and Δ denote the delay operator and the backward-difference operator, respectively,

$$z^{-1}\{y_k\} \equiv y_{k-1} \text{ and } \Delta y_k \equiv (1 - z^{-1})y_k = y_k - y_{k-1} \quad (2)$$

where y_k, u_k, w_k, e_k are the output, the control, the measurable disturbance and an assumed white noise at time k, respectively. The indices du and dw represent the minimum delays associated with the input and the disturbance, respectively.

The polynomials A, B and C are given by:

$$A(z^{-1}) = 1 + a_1 z^{-1} + \cdots + a_n z^{-n}$$
$$B(z^{-1}) = b_0 + b_1 z^{-1} + \cdots + b_{n-1} z^{-n+1}$$
$$C(z^{-1}) = c_0 + c_1 z^{-1} + \cdots + c_q z^{-q}$$

where n is the order of the plant model and q is the order of the disturbance process model.

For an implicit formulation, Eq. 1 can be transformed into a T-step ahead predictive model using the identity:

$$1 - z^{-T} = F(z^{-1})A(z^{-1}) + z^{-T}G(z^{-1}) \quad (3)$$

where F and G are of order $T - 1$ and $n - 1$, respectively. The parameter T is an integer larger than or equal to the control delay du and defines the prediction-horizon; Hiram (1983) has shown that the polynomials F and G can always be found for any T.

If Eq. 1 is multiplied by F and Eq. 3 is applied, the result is

$$y_k = y_{k-T} + G(z^{-1})\Delta y_{k-T} + H(z^{-1})\Delta u_{k-du} + D(z^{-1})\Delta w_{k-dw} + \Delta\eta_k \quad (4)$$

where $H(z^{-1}) = F(z^{-1})B(z^{-1})$ and $D(z^{-1}) = F(z^{-1})C(z^{-1})$. The residual is given by $\eta_k = F(z^{-1})e_k$.

The polynomials G, H and D are defined by:

$$G(z^{-1}) = \alpha_1 + \alpha_2 z^{-1} + \cdots + \alpha_n z^{-n+1}$$
$$H(z^{-1}) = \beta_1 + \beta_2 z^{-1} + \cdots + \beta_{n+T-1} z^{-n-T+2}$$
$$D(z^{-1}) = \delta_0 + \delta_1 z^{-1} + \cdots + \delta_{q+T-1} z^{-q-T+1}$$

Considering that in real plants both delays, du and dw, are unknown and may be time-varying, in the absence of other information it is the safest to use their minimum possible values in the model (Eq. 4). The minimum value for du is 1, since the control applied at time k cannot affect the output before time $k + 1$. On the other hand, because of the nature of sampled systems, a disturbance measured at time k can already have affected the output at time k. Thus the minimum value for dw is 0, so direct transmission between the disturbance w_k and the output y_k is a possibility. An unnecessarily large number of parameters may need to be estimated using these minimum values; larger values of du and dw can be used when appropriate.

The structure of Eq. 4 can be used for both estimation and control purposes.

Estimation

Using the definitions of the polynomials G, H and D, Eq. 4 can be expressed as a measurement equation:

$$y_k = y_{k-T} + \underline{\phi}_{k-T}\hat{\underline{\theta}} + \Delta\eta_k \quad (5)$$

where

$$\underline{\phi}_{k-T} = [\Delta y_{k-T} \ldots \Delta y_{k-n-T+1};\ \Delta u_{k-1} \ldots \Delta u_{k-n-T+1};\ \Delta w_k \ldots \Delta w_{k-q-T+1}]$$

$$\hat{\underline{\theta}}' = [\hat{\alpha}_1 \ldots \hat{\alpha}_n;\ \hat{\beta}_1 \ldots \hat{\beta}_{n+T-1};\ \hat{\delta}_0 \ldots \hat{\delta}_{q+T-1}]$$

The expected value of η_k given the available information at time $k - T$ is zero. The residual is a linear combination of values of the noise e from time $k + 1 - T$ to time k, which by definition are zero mean random variables uncorrelated with the information known at time $k - T$. Thus a recursive least squares (RLS) can be applied to Eq. 5 to estimate the α, β and δ parameters. A variable forgetting factor is used to automatically adjust the tracking of the parameters according to the characteristics of the process (Fortescue et al., 1981).

Control

The problems associated with minimum variance control, mainly due to excessive control action, are well known. The control action, however, can be easily detuned by using an extended horizon (EH) strategy. The minimization of the variance of the output error is carried out at a future time T, where T is an integer larger than or equal to the process delay du. In this way, the control action can be intuitively detuned without selecting a special control weight as required in generalized minimum variance (GMV) control, or locating closed-loop poles as in pole-placement control.

Using Eq. 5 to predict the output at time T, yields:

$$\hat{y}_{k+T} = Y_k + \sum_{i=1}^{T} \hat{\beta}_i \Delta u_{k-i+T} + \sum_{i=0}^{T-1} \hat{\delta}_i \Delta w_{k-i+T} \tag{6}$$

where Y_k represents a combination of known input/output data at time k and is given by:

$$Y_k = y_k + \sum_{i=1}^{n} \hat{\alpha}_i \Delta y_{k+1-i} + \sum_{i=1}^{n-1} \hat{\beta}_{T+i} \Delta u_{k-i} + \sum_{i=0}^{q-1} \hat{\delta}_{T+i} \Delta w_{k-i} \tag{7}$$

Y_k also represents the predicted value of y_{k+T} if $\Delta u_k = \Delta u_{k+1} = \ldots = 0$ and $\Delta w_k = \Delta w_{k+1} = \ldots = 0$ (i.e., constant control action and no new disturbances).

To comply with the extended horizon control objective, the following equation must be satisfied for the future control sequence:

$$\sum_{i=1}^{T} \hat{\beta}_i \Delta u_{k-i+T} = y^*_{k+T} - Y_k - \sum_{i=0}^{T-1} \hat{\delta}_i \Delta w_{k-i+T} \tag{8}$$

where y^*_{k+T} represents the set point or reference value at time $k + T$.

The future control sequence $\{\Delta u_i\}_{i=k}^{k+T-1}$ can be chosen according to the strategies discussed by Ydstie et al. (1985) and Pérez-Correa (1987).

Two possible strategies are:

i) Choose equal control increments over the horizon T (linear control action). From Eq. 8, the increments are defined by:

$$\Delta u_k = \left[y^*_{k+T} - Y_k - \sum_{i=0}^{T-1} \hat{\delta}_i \Delta w_{k-i+T} \right] \Bigg/ \sum_{j=1}^{T} \hat{\beta}_j \tag{9}$$

ii) Concentrate the control effort on the first increment and set to zero the rest of the control increments (constant control). The first input increment of the sequence is given by:

$$\Delta u_k = \left[y^*_{k+T} - Y_k - \sum_{i=0}^{T-1} \hat{\delta}_i \Delta w_{k-i+T} \right] \Bigg/ \hat{\beta}_T \tag{10}$$

Many other strategies exist as well (Ydstie et al., 1985; Pérez-Correa, 1987).

Correspondingly, future values of the disturbance w, which in general are not known, also appear in Eqs. 8, 9 and 10. Several hypotheses can be made about these future disturbances. Some reasonable assumptions are:

a) The disturbance is mainly deterministic, changing by steps and remaining constant for long periods. Then, it can be postulated, $\Delta w_{k+1} = \Delta w_{k+2} = \ldots = \Delta w_{k+T} = 0$, so that the best estimate of the future input disturbances is reduced to:

$$\sum_{i=0}^{T-1} \hat{\delta}_i \Delta w_{k-i+T} = 0 \tag{11}$$

This is a typical type of disturbance in chemical plants.

b) The measurable disturbance is likely to be a white noise process. Then, all the future values are expected to be zero. In this case, $\Delta w_{k+1} = -w_k$ and $\Delta w_{k+2} = \Delta w_{k+3} = \ldots = \Delta w_{k+T} = 0$, so that:

$$\sum_{i=0}^{T-1} \hat{\delta}_i \Delta w_{k-i+T} = -w_k \hat{\delta}_{T-1} \tag{12}$$

c) The future disturbance is likely to continue as a ramp function:

$$\Delta w_{k+i} = \Delta w_k \qquad i = 1 \ldots T$$

The expression for the future disturbances is, in this case, given by:

$$\sum_{i=0}^{T-1} \hat{\delta}_i \Delta w_{k-i+T} = \Delta w_k \sum_{i=0}^{T-1} \hat{\delta}_i \tag{13}$$

It is worth noting that in any case the control law is implemented in a receding-horizon form, i.e., only the first control increment of the sequence is computed each sample time.

Any adaptive algorithm needs to be provided with some design parameters and initial values. The most important design parameters are:

a) Time horizon T which defines the future time at which the variance of the output is minimized.

b) Orders of the polynomials of the assumed plant model; n for the process model and q for the disturbance model.

c) Information content of the RLS parameter estimator (Σ_0), which defines the rate of adaptation of the self-tuner (see Fortescue et al., 1981).

The initial values are much less critical to algorithm performance, which include:

d) Initial covariance matrix of the parameter estimator, usually set to a diagonal with very large elements.

e) Initial parameter estimates, generally set to zero (taking care not to divide by zero initially).

Depending upon individual circumstances, any feedback-control strategy defined by Ydstie et al. (1985) and Pérez-Correa (1987) can be used together with Eq. 11, 12 or 13 to get a feedback-feedforward control law. As an example, a combination of a strategy which considers constant control input during the horizon (Eq. 10) and assumes deterministic disturbances (Eq. 11) are used in the algorithm below.

FB/FF EH algorithm

i) *Prediction step:*

$$\hat{y}_k = y_{k-T} + \underline{\phi}_{k-T}\underline{\hat{\theta}}_{k-1}$$

where

$$\underline{\phi}_{k-T} = [\Delta y_{k-T}, \ldots, \Delta y_{k-n-T+1};\quad \Delta u_{k-1}, \ldots, \Delta u_{k-n-T+1};\quad \Delta w_k, \ldots, \Delta w_{k-q-T+1}]$$

$$\underline{\hat{\theta}}'_{k-1} = [\hat{\alpha}_1, \ldots, \hat{\alpha}_n;\quad \hat{\beta}_1, \ldots, \hat{\beta}_{n+T-1};\quad \hat{\delta}_0, \ldots, \hat{\delta}_{q+T-1}]$$

ii) *Estimation* (a RLS estimator with variable forgetting factor is used to update the parameters, $\hat{\theta}_k$):

Let

$$v_k \equiv y_k - \hat{y}_k$$

then,

$$s_k = \underline{\phi}_{k-T}P_{k-1}\underline{\phi}'_{k-T}$$

$$n_k = 1 - s_k - v_k^2 \Big/ \sum_0,$$

and

$$\lambda_k = (n_k + n_k^2 + 4\, s_k)/2 \qquad \text{(variable forgetting factor)}$$

$$\underline{K}_k = P_{k-1}\underline{\phi}'_{k-T}/(\lambda_k + s_k) \qquad \text{(estimator gain)}$$

$$\underline{\hat{\theta}}_k = \underline{\hat{\theta}}_{k-1} + \underline{K}_k v_k \qquad \text{(parameter update)}$$

$$P_k = [P_{k-1} - \underline{K}_k\underline{K}'_k(\lambda_k + s_k)]/\lambda_k \qquad \text{(covariance update)}$$

iii) *Control:*

$$\Delta u_k = (y^*_{k+T} - Y_k)/\hat{\beta}_T$$

where

$$Y_k = y_k + \sum_{i=1}^{n} \hat{\alpha}_i \Delta y_{k+1-i} + \sum_{i=1}^{n-1} \hat{\beta}_{T+i}\Delta u_{k-i} + \sum_{i=0}^{q-1} \hat{\delta}_{T+i}\Delta w_{k-i}$$

iv) *go to (i)*

Note that the actual code of the RLS routine in (ii) was implemented using Peterka's square root approach (Peterka, 1975).

Results and Discussion

Simulations

The control algorithm described above has been tested through simulations to examine many of its characteristics including:

(i) Detuning properties of the FB/FF self-tuner

(ii) Influence of direct disturbance transmission

Detuning the Control Action. A serious potential problem with feedforward control is the strong control action usually required to obtain complete disturbance rejection. This phenomenon is shown in a simulation with the following third-order process:

$$y_k = 1.2\, y_{k-1} - 0.2\, y_{k-2} - 0.02\, y_{k-3} - u_{k-1} - 0.95\, u_{k-2} - 0.02\, u_{k-3} - w_{k-1} - 0.5\, w_{k-2} - 0.08\, w_{k-3} \quad (14)$$

The measured disturbance w is a square wave which varies between 0.95 and 1.25.

Figure 1 compares the performance of the controlled system when minimum-variance ($T = 1$) and extended-horizon ($T = 3$) FB/FF strategies are used. The minimum-variance control achieves perfect regulation; however, the control u rings excessively. On the other hand, by increasing the control horizon to 3, the ringing in the control is avoided with minimum effects in the output. In the same way, saturation can be avoided by increasing the control horizon. Thus, the same desirable detuning property of the FB/EH controller carries over to the FB/FF EH controller. Furthermore, as has been shown by previous workers (Ydstie et al., 1985; Hiram and Kershenbaum, 1985), the detuning effect of the extended-horizon strategy allows the controller to cope with nonminimum phase processes.

Direct Disturbance Transmission. Because of the discrete nature of the sampling process, a disturbance measured at time k may have already affected the output at time k. Because of

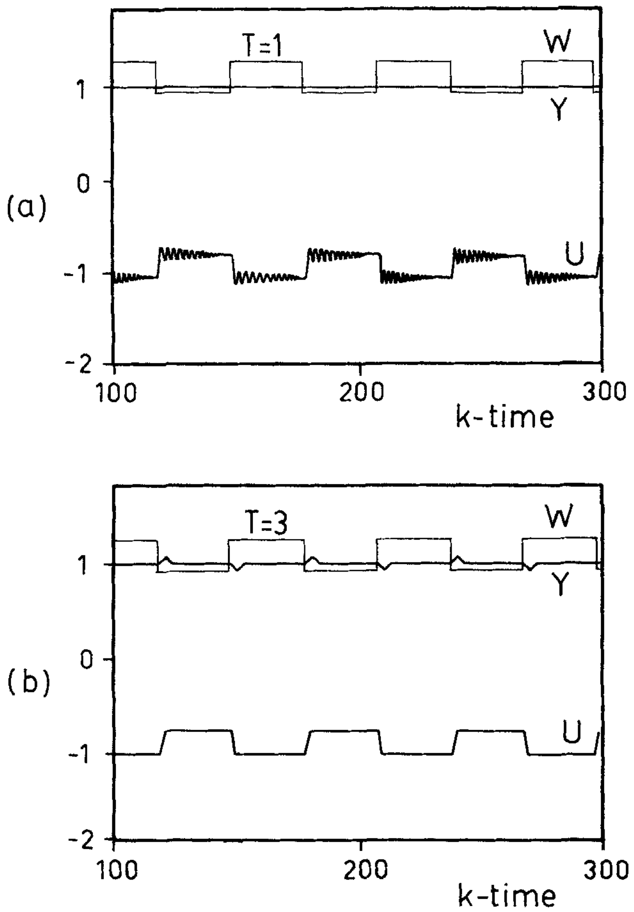

Figure 1. Comparison of FB/FF controller performance for step changes in measurable disturbance, *w*: (a) minimum variance (*T* = 1); (b) extended horizon (*T* = 3).

this, Eq. 5 considers direct transmission between the measured disturbance w_k and the output y_k. The importance of this effect is shown in Figure 2a where direct transmission is not modeled, that is, dw is taken as 1 in Eq. 4. The following first-order linear model was used in the simulations:

$$y_k = 0.64\, y_{k-1} + 0.5\, u_{k-1} + w_k \tag{15}$$

The measured disturbance w_k is a square wave that takes the values 1 and 1.2, and immediately affects the output y_k.

It can be seen in Figure 2a that at the beginnning the control is good, but eventually variations in the parameter estimates make the algorithm explode (Pérez-Correa, 1987). The same model of Eq. 15 was controlled with the regulator which allows for direct transmission between the measured disturbance and the output ($dw = 0$ in Eq. 4). Figure 2b shows that good control is achieved despite the fact that the disturbance immediately affects the output. The parameter estimates remained constant and were not influenced by direct transmission of the disturbance (Pérez-Correa, 1987). Similar results were obtained with many other simulations. Setting $dw = 0$ in Eq. 4 can improve the robustness of the self-tuner when sample times are large and the effects of disturbances are likely to be felt between samples. The price paid is that an additional parameter must be estimated.

Many other simulation tests were carried out to investigate the robustness of the algorithm to choice of parameters, nonlinearities, etc. These are discussed in detail elsewhere (Pérez-Correa, 1987); instead, the remaining emphasis here is concentrated on the experimental tests performed.

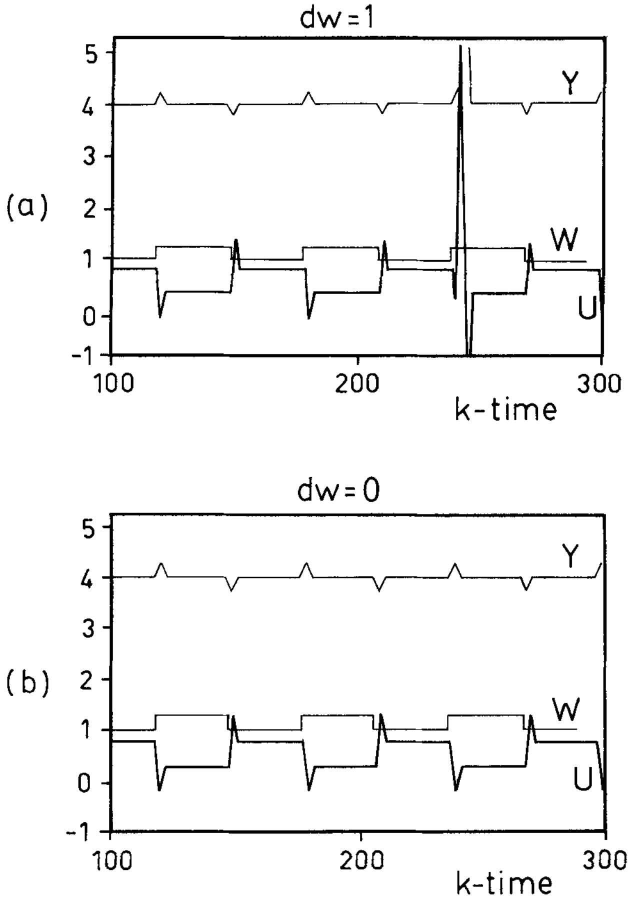

Figure 2. Effects of direct disturbance transmission: (a) possibility ignored (dw = 1); (b) possibility accounted for (dw = 0).

Plant experiments

Numerous tests of self-tuning algorithms have been performed on the computer-controlled pilot plants at Imperial College. These have been carried out on pressure, level, flow and composition loops within the pilot plant, which separates CO_2 from N_2 via absorption and desorption using an aqueous MEA solution (Kershenbaum and Fortescue, 1981; Ydstie et al., 1985; Hiram and Kershenbaum, 1985).

The pilot plants are interfaced to the computer system via an Analogics 4400 unit. An IBM series 1 minicomputer links the Analogics unit with the mainframe computer (IBM 4341), where all of the control calculations are performed. IBM's control package, ACS (Advanced Control System) enables the monitoring and control of the plant with built-in standard strategies. A special interface enables the coding of advanced control strategies in Fortran or Pascal.

The level control loops of the absorber of the CO_2 pilot plant were used as a convenient, but stringent, test for the proposed FB/FF self-tuner, Figure 3. Normally, the level of the liquid in the bottom of the absorption column is controlled by the outlet valve (configuration 1 in Figure 3). In this case, the time delay between the control and output is less than 10 seconds. In an alternative scheme, an artificially difficult configuration can be constructed, where the level is controlled by the inlet valve (configuration 2 in Figure 3). This abnormal arrangement introduces a large time delay which results from the trickling flow down the packed column. Furthermore, the time delay is highly dependent on the liquid flow rate, the gas flow rate up the column, and the previous state of the column. This delay can vary between 30 and 90 seconds.

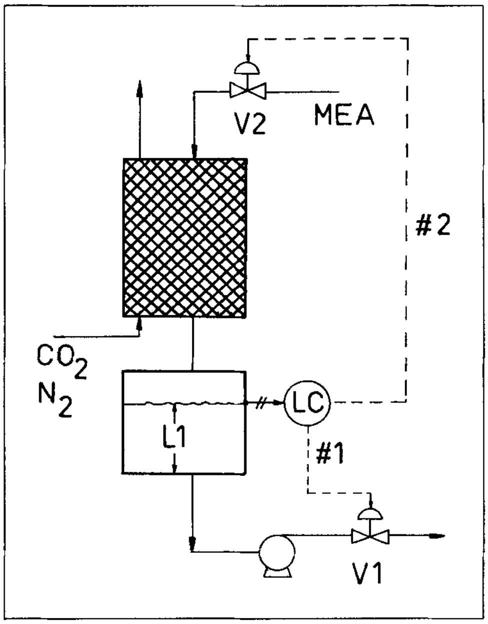

Figure 3. Level control configurations in CO_2 absorption plant.

The experimental study focussed on the following issues:

- Comparison of a FB/FF controller with a purely FB one
- Influence of initial settings on the self-tuner performance
- Comparison of alternative control strategies

In the experimental results shown, a measure of the feedforward compensation is given by the sum:

$$\Sigma\delta = \delta_T + \delta_T + 1 + \cdots + \delta_{T+q-1} \qquad (16)$$

where the δ parameters are the appropriate ones for the control law used (Eqs. 6–8).

Similarly, a measure of the feedback action is given by the controller gain, which depends on the control strategy selected and is related to the reciprocal of the β parameters as shown in Eqs. 9 and 10.

In all the experimental work, we followed the normal industrial practice of limiting the incremental change of the control signal to 20% of the input range, despite the control action called for by the algorithm.

Feedback/Feedforward vs. Purely Feedback Control. Figure 4 compares the behavior of a FB/FF self-tuner with a purely FB one; an equal-increments control strategy (Eq. 9) was applied in both cases. In these experiments, the level was controlled with the inlet valve (configuration #2) and the measured disturbance is the outlet flow. This control configuration, as mentioned before, is characterized by a large and varible control delay. The information length Σ_0 for adaptation of the estimator was set to 0.1 and the time horizon T to 5 in the FB/FF experiment; in the FB-only case, marginally better results were obtained when Σ_0 was set to 0.01 and T was set to 4. The model order and the sample time were the same in both experiments.

In the experiments, the set point was kept constant but load disturbances (changes in outlet flow rate of up to 20%) were introduced every 5 minutes. The performance of the FB/FF algorithm (Figure 4a) is almost perfect; the deviations from the set point are inevitable because of the long time delay (≈ 1 minute) between the control action and the output. Also plotted in Figure 4a is the reciprocal of the controller gain. This remains nearly constant despite the load disturbances, indicating only small changes in the estimated parameters, β.

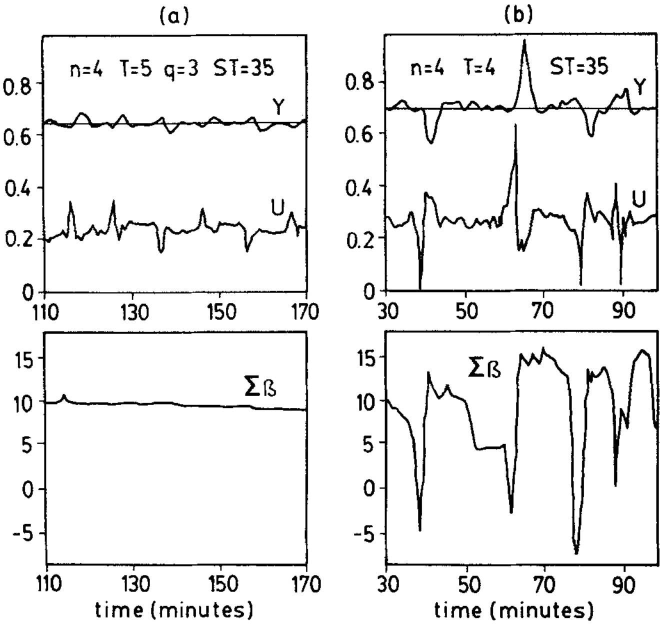

Figure 4. Self-tuning level control using configuration #2: (a) FB/FF control; (b) FB control only.

In contrast, the FB-only algorithm (Figure 4b) has occasional large excursions in the output. This is caused by the relatively low robustness of the feedback-only controllers in systems with such large (and variable) time delays. It is clear that some of the parameters (and, hence, the controller gain) fluctuate drastically; this leads to situations in which the gain becomes infinite (near zero values of the reciprocal gain) and even changes sign, thereby causing temporary excursions from the set point. Previous work (Hiram and Kershenbaum, 1985) had shown that these excursions in the FB algorithm could be made smaller, but only by detuning the controller still further and making the performance much more sluggish.

Note that no feedforward model had to be provided for the relationship between the measured disturbance and the output; only an estimate of the model order was required. In this way, the main drawback of feedforward (nonadaptive) control, namely the requirement of the availability of a good process model, is avoided; the necessary I/O relationship is estimated on-line. These results are typical of many others obtained (Pérez-Correa, 1987).

Choice of Parameters. It has often been reported (Morris et al., 1985; Pérez-Correa, 1987; Lim et al., 1987) that self-tuning controllers can experience deteriorated performance when subjected to large and frequent load changes. In the case of positional algorithms, such changes in load cause variations in the bias term of the model and lead to poor performance as discussed by Clarke et al. (1983). Moreover, those authors suggest that a simple incremental algorithm will not be optimal in systems with large or variable time delays.

A similar deterioration in the performance of the proposed FB/FF self-tuning algorithm was noted under certain conditions when the controlled system was subjected to such large and frequent load disturbances. The poor performance was accompanied by (and probably caused by) a drift in the parameter estimates which lead to large values of the controller gain.

Figures 5 and 6 show typical experimental results obtained when different values of the algorithm parameters were used. In these experiments, the liquid level in the absorption column is controlled by the outlet valve position (configuration #1) and the measured disturbance is the inlet flow rate. There is clearly a large and uncertain time delay before the disturbance affects the output; hence, one would expect the estimated feedforward model to be subject to some significant uncertainty. The measurable disturbance, the inlet flow rate changes by a factor of 2 every 5 minutes; furthermore, this change causes a 25–50% change in the delay time. The feedback control in this case is trivial (since there is virtually no delay between the control and the output); hence, any deterioration in performance can be attributed to misinformation provided by the feedforward component of the algorithm.

Figure 5a illustrates the unsatisfactory performance of the algorithm when relatively small values are chosen for the order of the disturbance model ($q = 5$) and the time horizon ($T = 3$); the sample interval in these experiments is 15 seconds. The poor performance is caused by a drift in the model parameter estimates as reflected by the drift in the reciprocal gain of the controller as shown in Figure 5b.

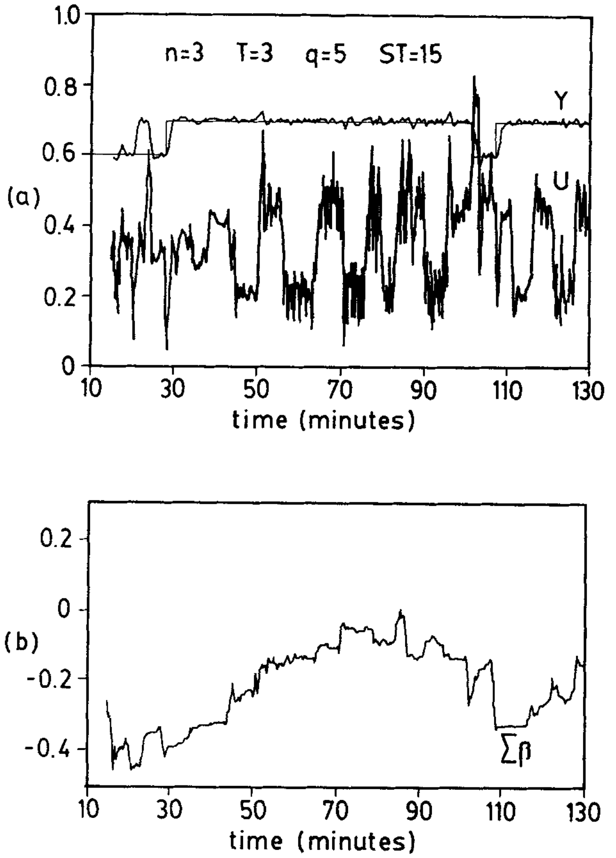

Figure 5. Self-tuning level control using configuration # 1 and insufficient data acquisition: (a) output and control variables; (b) reciprocal controller gain.

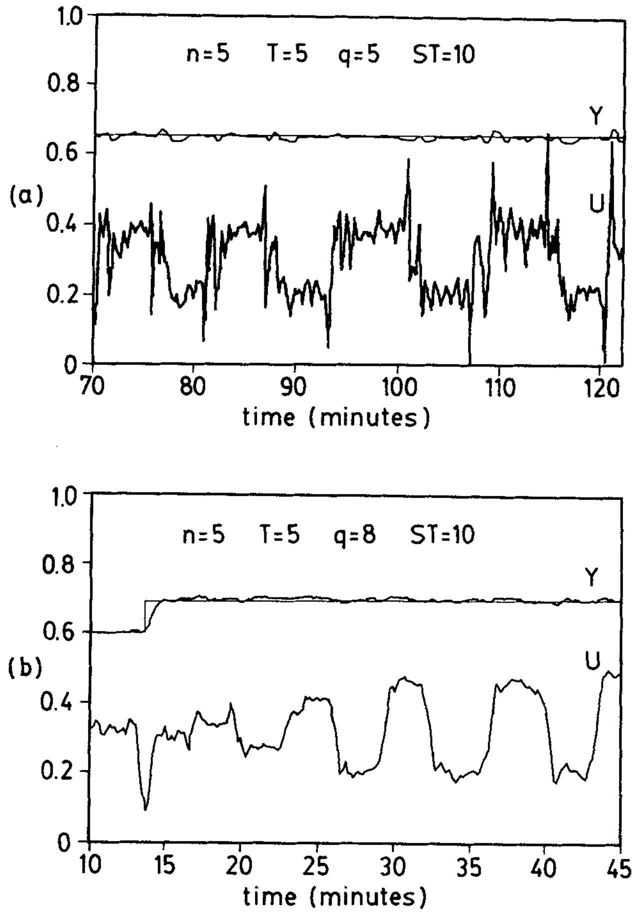

Figure 6. Self-tuning level control using configuration # 1 and increased data acquisition: (a) $n = 5$, $q = 5$; (b) $n = 5$, $q = 8$.

The situation can be significantly improved if a larger time horizon T or disturbance model order q are used. Both of these serve to provide additional input/output data for the feedforward estimator. In either case, an increased number of coefficients must be estimated; if the time horizon is increased, the control action is also made somewhat more sluggish. Typical results are shown in Figure 6.

With a disturbance model order of $q = 5$ (Figure 6a), uncertain parameter estimates lead to oscillatory control although some improvement is seen relative to Figure 5. In Figure 6b, the model order of the disturbance process has been increased to $q = 8$, the result is a stable and smooth control action and a near-perfect regulation of the liquid level. In this case, the data vector contains sufficient information about the disturbance so that the parameter estimator is now able to produce a good model of the disturbance. The parameter estimator is even able to provide an accurate estimation of the disturbance delay. Equation 4 confirms that, for $dw = 0$, successful estimation of the feedforward parameters δ requires $q + T$ to be greater than the actual disturbance delay.

Control Strategies. The selection of a suitably detuned control strategy is also important in the final performance of a self-tuning algorithm. In particular, all the above experiments employed equal-increments extended-horizon control (Eq. 9), in which the future control action is assumed to increase linearly over the control horizon. It was felt that this strategy would be more robust in the incremental algorithm as no abrupt change in the closed loop is artificially introduced by, for example, concentrating the control effort in the first increment. Indeed, the equal-increments strategy should be robust in the nonadaptive case since the called-for control action is more gradual and the controller gain is the summation of various terms, *viz,*

$$1 \Big/ \sum_{i=1}^{T} \beta_i.$$

However, a problem arises in the adaptive case where the β parameters must be estimated. It has been observed in simulations and pilot-plant experiments that the first $\hat{\beta}$ estimates [$\hat{\beta}_1$ to $\hat{\beta}_{T-1}$] in the parameter vector are slow to converge, are easily detuned and sometimes take a sign contrary to the sign of the true process; this is because these parameter estimates which (depending upon the delay time) are close to zero, generally receive poor excitation. Under these conditions, the adaptive control gain reaches very large values or even takes on the opposite sign. This contributes to the phenomenon shown by Figure 5.

The behavior of some of these $\hat{\beta}$ estimates can be appreciated in the experiment shown in Figure 7. In this experiment the level is controlled by the outlet valve (configuration #1) and the sys-

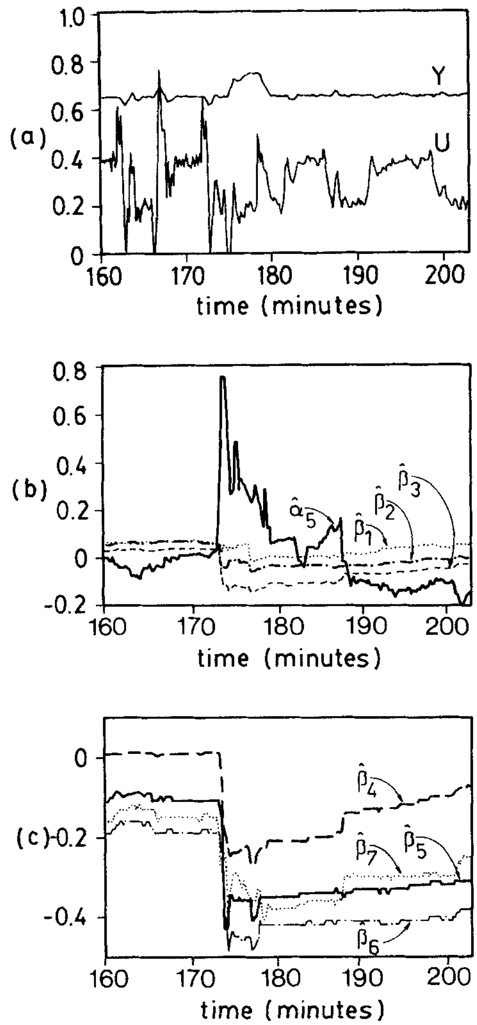

Figure 7. Self-tuning level control using configuration #1, $n = 5$, $q = 8$: (a) output and control variables; (b) (c) evolution of parameter estimates.

tem is continuously affected by measured deterministic disturbances provided by inlet flow step changes. A constant control strategy (Eq. 10) was employed, which is equivalent to concentrating the incremental control effort in the first increment. For $t < 160$, the algorithm was fed with corrupted and inaccurate data. In this situation, the parameter estimates converged to unsuitable values. Figure 7 shows the evolution of the plant after the resumption of correct data acquisition. It is interesting to see in Figures 7b and 7c the behavior of the $\hat{\beta}$ estimates. After the recovery, the first $\hat{\beta}$ estimates ($\hat{\beta}_1$, $\hat{\beta}_2$, $\hat{\beta}_3$ and $\hat{\beta}_4$) have a positive sign despite the fact that the controller gain should be negative. The rest of the $\hat{\beta}$ estimates always remain negative. This is typical of the observation that there is more uncertainty associated with the first $(T - 1)$ parameters (*some* should actually be zero because of the time delay) and hence they should be given little or no weight in the controller. Had the equal-increments control strategy been employed (giving equal weight to all T first $\hat{\beta}$ estimates), the resulting gain would have been positive, giving positive feedback. This behavior of the $\hat{\beta}$ estimates has been observed in many other experiments and simulations. Any adaptive control strategy which uses these estimates is likely to present problems. For this reason, the equal increments strategy was abandoned and replaced by the more robust constant control strategy.

To test the robustness of the constant control strategy, the more difficult level-loop (configuration #2) was controlled with different initial settings. The disturbance is produced by outlet flow step changes every 5 minutes. Figures 8a and 8b present a similarly stable response of the controlled system for rather different algorithm parameters. The feedforward parameter $\Sigma\delta$ and the reciprocal feedback gain remained stable in both experiments (Pérez-Correa, 1987).

Conclusions

A FB/FF SISO self-tuning algorithm was proposed and tested with systems characterized by large and frequent load changes and time delays. The algorithm was implemented in an implicit-incremental form. The controller included direct transmission between the measured disturbance and the measured output to account for intersample perturbations. The FF control law assumes that disturbances change by infrequent steps; however, other kind of disturbance models may also be considered. The FB/FF self-tuner has shown a much better performance than a FB-only algorithm at very little extra cost, namely, a slower convergence rate due to the increased number of parameters which need to be estimated. Like any other self-tuning controller, the algorithm proposed in this work requires the selec-

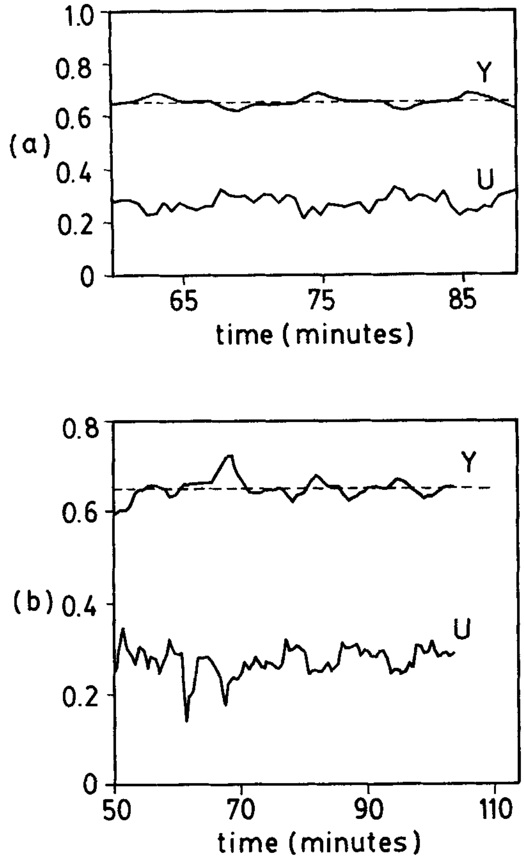

Figure 8. Self-tuning level control using configuration #2: (a) $n = 3$, $q = 1$, $T = 4$; (b) $n = 4$, $q = 3$, $T = 5$.

tion of a set of initial parameters. The exact values of these parameters are generally not crucial in obtaining a good performance; the most important of these parameters are chosen as follows:

- *Time-Horizon (T).* When FB/FF controllers are employed perfect regulation can be achieved, but undesirably strong control action may be required. In this case, the horizon T can be increased to detune the control, typically to one or two sample times more than the maximum anticipated time delay. It has been observed that, once the control has been detuned, increasing the horizon even more has little effect on the control performance.
- *Model Orders (n, q).* The SISO FB/FF controller requires the selection of two model parameters, one for the control/output transfer function (n), another for the disturbance polynomial (q). If the plant has no time delay, the selection of n and q is not at all sensitive. On the other hand, if the disturbance is measured with a delay, the selected parameter q must be large enough to ensure that significant information is included in the data vector of the parameter estimator. The value of q must be such that $q + T$ is greater than the maximum anticipated disturbance delay.
- *Information Length or Content (Σ_0).* This parameter is defined as the product of the process noise variance and the number of sample intervals contained in the data window (Ydstie et al., 1985). However, Σ_0 can also be considered as just a tuning parameter which controls the rate of adaptation. It has been observed that too small a value produces undesirable "burst" effects on the estimates and the control. On the other hand, too large a value makes the estimator insensitive so that the algorithm cannot adapt to plant changes. Nevertheless, the selection of the information length is very robust: it can be safely selected in a range of two orders of magnitude. Normally, the information length is selected between 20 and 200 times the estimated noise variance.

The resulting algorithm incorporates the main advantages of FF control, i.e., compensation of the disturbance before it affects the output and the possibility of complete disturbance rejection. Moreover, the proposed self-tuner eliminates the need for an accurate model of the system and its time delays. It also provides a simple way to detune the control action and, thereby, improve robustness and avoid controller saturation.

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Notation

A, B, C, D = polynomials in discrete model
a, b, c = model parameters
du, dw = minimum delay time in control, disturbance
e = white noise disturbance
F, G, H = polynomials in discrete model
k = time interval
K = estimator gain
n = model order for control input
P = estimator covariance
q = model order for disturbance input
s = weighted squared data vector
T = control horizon
u = control input
v = model prediction error
w = disturbance input
y = output
Y = predicted future output (eq. 7)
z^{-1} = backward shift operator

Greek letters

α, β, δ = model parameters
Δ = difference operator
η = predictive model residual
θ = parameter vector
λ = variable forgetting factor
Σ_0 = information content of estimator
ϕ = data vector

Superscripts

$\hat{}$ = predicted/estimated values
$*$ = set point

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